International Journal of Theoretical Physics, Vol. 31, No. 6, 1992

Multiphase Percolation and Josephson Model for High-Temperature Superconductivity

El-Sayed M. El-Horbaty[1,3] and E. Ahmed[2,3]

Received January 20, 1991

A four-phase percolation problem is used to simulate the Josephson model for high-temperature superconductivity both as a four-phase and a three-phase system. We implement the method on an IBM PC microcomputer using PASCAL language.

Percolation theory (Stauffer, 1985) has many applications to critical phenomena. Recently it has been used (Meilikhov and Gershanov, 1989; Clem, 1988) in a model of high-temperature superconductivity (HTSC) (Bednorz and Muller 1986) called the Josephson model. This model depends on the granular structure of HTSC. In this model HTSC is represented by superconducting grains. These grains are connected by links whose conductivities depend on their lengths. This corresponds to a percolation problem where the randomness is not the existence of a bond (or a site), but a property of the bond (or the site).

This can be simulated as follows: Distribute superconducting sites randomly. The properties of the sites between two superconducting sites depend on their label numbers. For simplicity we assume that there are three types (or two) of these intermediate sites. Thus, we are faced with a four- (or three-) phase percolation problem.

To calculate the conductivity, we use Grey Scaling (Ahmed and Tawansi, 1991, and El-Misiery *et al.*, 1991, and the references therein). For completeness, we review this method here. The idea of scaling in critical phenomena depends on the hypothesis that near the critical threshold the

[1]Mathematics Department, Faculty of Science, Ain Shams University, Egypt.
[2]Mathematics Department, Faculty of Science, Mansoura University, Egypt.
[3]Present address: Mathematics and Computer Science Department, Faculty of Science, Emirates University, Al Ain, U.A.E.

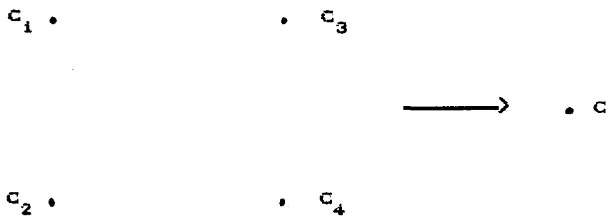

Fig. 1. The reduction used in equation (1).

correlation length is so large that the system is self-similar. This enables us to lump small units together to build larger units without changing the system significantly. We apply this idea to calculate the conductivity of a square grid network (square lattice) as follows: Consider a square grid network with node conductivities c_1, c_2, c_3, and c_4 (see Figure 1). Assume that the current direction is vertically downward, so that nodes 1 and 2 are

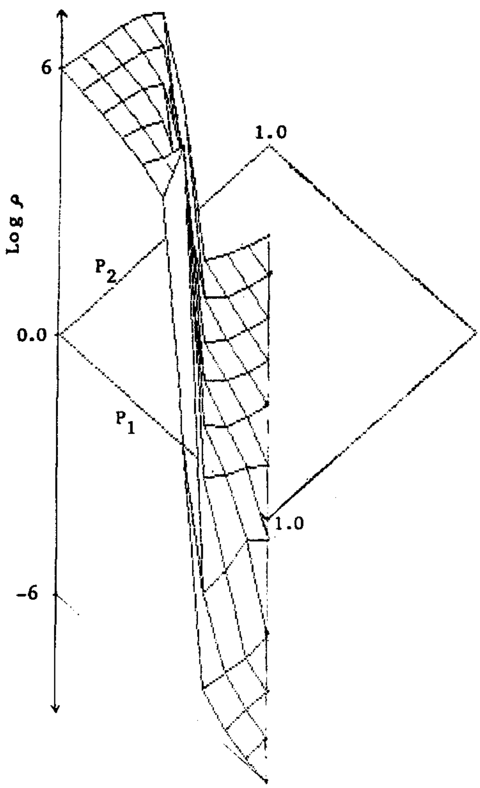

Fig. 2. The decimal logarithm of the resistivity as a function of P_1 and P_2 for $P_3 = 0.0$.

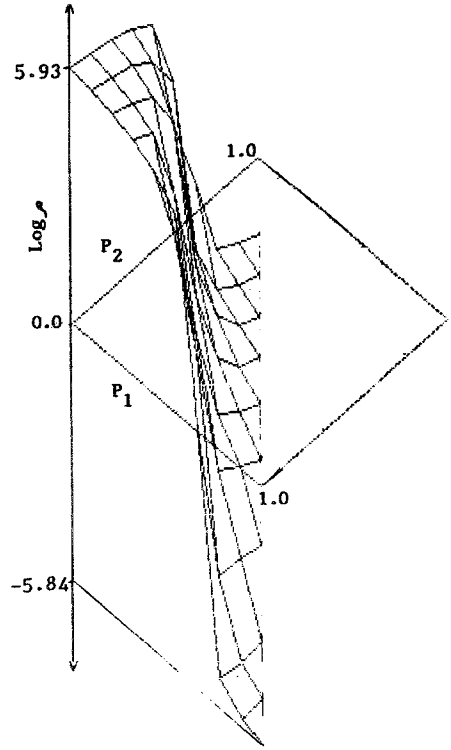

Fig. 3. The decimal logarithm of the resistivity as a function of P_1 and P_2 for $P_3 = 0.1$.

in series, and nodes 3 and 4 are in series. Then this node (cell) is replaced by an equivalent supernode whose conductivity c is

$$c = \frac{c_1 c_2}{c_1 + c_2} + \frac{c_3 c_4}{c_3 + c_4} \tag{1}$$

We carry on this lumping process iteratively until the whole network is equivalent to a single super-super-···-supernode whose conductivity is equal to the conductivity of the random network.

Theoretically, Grey Scaling is a real-space renormalization implemented on a computer. Using this method, all the known results in site percolation about electrical conductivity, the critical concentration P_c, and the conductivity exponents in all dimensions $2 \leq d \leq 6$ have been reproduced. Furthermore, this method avoids having to solve Kirchoff's equations at each node. Consequently, it is easily implemented on small computers, including PCs.

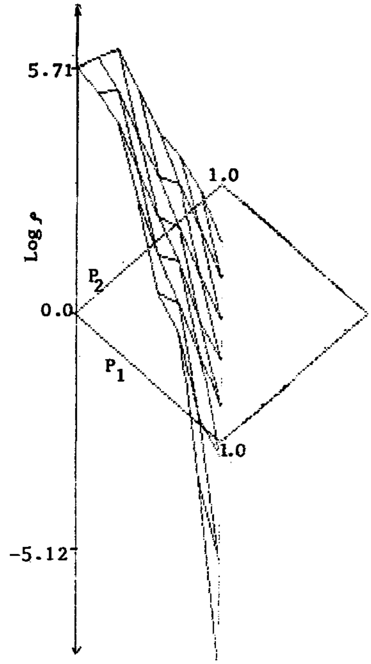

Fig. 4. The decimal logarithm of the resistivity as a function of P_1 and P_2 for $P_3 = 0.3$.

Recently (El-Misiery *et al.*, 1991) the conductivity of three-phase systems has been simulated. In this paper we find the conductivity of four-phase systems. We assume the existence of four types of conductors occupying randomly the nodes of a square grid network. Let P_i, $i = 1, 2, 3, 4$, be the probability of the ith phase; hence

$$P_1 + P_2 + P_3 + P_4 = 1 \tag{2}$$

and the distribution function in conductivity σ is

$$g(\sigma) = P_1\delta(\sigma - \sigma_1) + P_2\delta(\sigma - \sigma_2) + P_3\delta(\sigma - \sigma_3) + P_4\delta(\sigma - \sigma_4) \tag{3}$$

In our simulation we set $\sigma_1 = 10^6$, $\sigma_2 = 10^2$, $\sigma_3 = 10^{-2}$, and $\sigma_4 = 10^{-6}$. The critical surfaces are

$$P_1 = P_c = 0.6, \qquad P_4 = 1 - P_c \tag{4}$$

which agree with the two- and three-phase results in two dimensions (El-Misiery *et al.*, 1991; Kugot and Straley, 1979). There are three regions for the conductivity. The first region is given by $P_1 \geq 0.6$, in which the bulk conductivity σ is of order σ_1, i.e., $\sigma = O(\sigma_1)$. The second region is $P_4 \geq 0.4$,

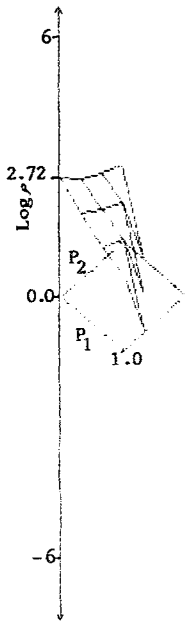

Fig. 5. The decimal logarithm of the resistivity as a function of P_1 and P_2 for $P_3 = 0.7$.

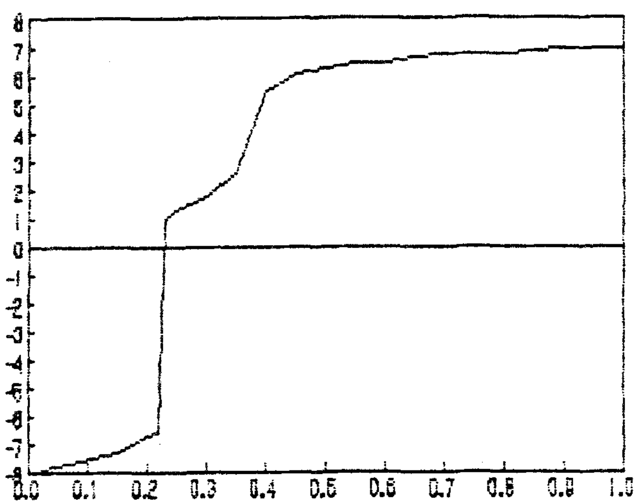

Fig. 6. The decimal logarithm of the resistivity of the four-phase simulation for the Josephson model vs. the concentration P of the most insulating phase.

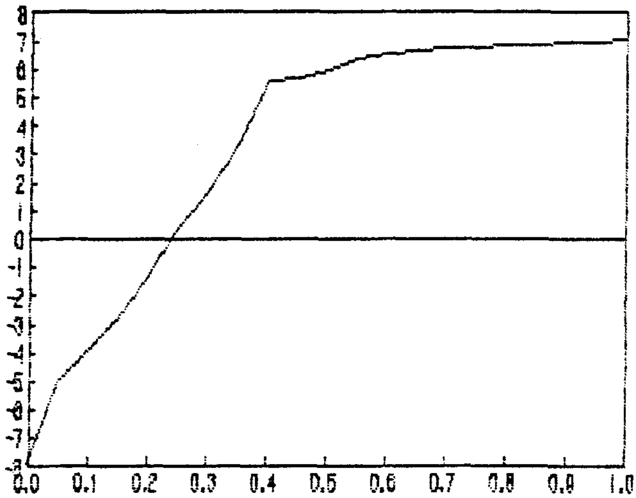

Fig. 7. The decimal logarithm of the resistivity of the three-phase simulation for the Josephson model vs. the concentration P of the most insulating phase.

in which $\sigma = O(\sigma_4)$. The third region is given by $P_1 < 0.6$ and $P_4 < 0.4$ and in which $\sigma = O(\sigma_2)$ or $\sigma = O(\sigma_3)$. It is surprising that there is no surface separating those two values. The plateau noticed in the three-phase system (El-Misiery *et al.*, 1991) still exists in the four-phase system and its defining relations are

$$P_1 < P_c \quad \text{and} \quad P_4 < 1 - P_c \tag{5}$$

In Figures 2–5 we present the decimal logarithm of the resistivity log ρ ($\rho = 1/\sigma$) as a function of P_1 and P_2, with P_3 set equal to 0.0, 0.1, 0.3, and 0.7, respectively.

Having solved the four-phase problem, we can simulate the Josephson model for HTSC. We present two simulations. In the first we assume the existence of four phases, one of them a superconductor with conductivity $\sigma_1 = 10^8$; the other three phases have conductivities $\sigma_2 = 1000$, $\sigma_3 = 10^{-1}$, and $\sigma_4 = 10^{-7}$, respectively. We assume that P is the probability for the most insulating phase (the fourth). The distribution function is given by

$$g(\sigma) = P\delta(\sigma - \sigma_4) + (P^{0.6} - P)\delta(\sigma - \sigma_3) + (P^{0.3} - P^{0.6})\delta(\sigma - \sigma_2) + (1 - P^{0.3})\delta(\sigma - \sigma_1) \tag{6}$$

This distribution function has been derived using the theoretical results for the Josephson model. The results for the logarithm of the resistivity vs. the probability P are given in Figure 6. The critical concentration for the superconductor–nonsuperconductor transition is given by

$$P = 0.22 \tag{7}$$

which agrees with the theoretical results.

A simpler three-phase system that simulates the Josephson model can be obtained from the previous model by removing the second phase. The results are shown in Figure 7.

We implemented our method on an IBM PC microcomputer using PASCAL language. All the simulations were done on 64×64 square grid network and each result was averaged ten times.

REFERENCES

Ahmed, E., and Tawansi, A. (1991). *Physics Letters A*, **153**, 483.

Bednorz, J. G., and Muller, K. A. (1986). *Zeitschrift für Physik B*, **64**, 189.

Clem, J. R. (1988). *Physica C*, **153–155**, 50.

El-Misiery, A., Ahmed, E., and Tawansi, A. (1991). *Physica B*, **168**, 128.

Kugot, P., and Straley, J. (1979). *Journal of Physics C*, **12**, 1.

Meilikhov, E., and Gershanov, Yu. (1989). *Physica C*, **157**, 413.

Stauffer, D. (1985). *Introduction to Percolation Theory*, Taylor and Francis, London.